The Non-existence of a Global Supremum in Collatz Trajectories via Infinite Iterative Increments

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Abstract

This article examines the unbounded nature of growth within Collatz trajectories by focusing on the non-existence of a global supremum. We demonstrate that specific structured integers of the form $n = 2^k - 1$ allow for the direct calculation of iterations that increment towards infinity. Because these initial steps guaranty sustained growth as k increases to infinity, the sequence evades any proposed upper bound, illustrating that height is limited only by the complexity of the starting value.

Keywords: Collatz Conjecture, Trajectory Growth, Global Supremum, Infinite Increments, $2^k - 1$ Sequences, Iterative Calculation.

Introduction

A fundamental challenge in the Collatz conjecture is the extreme variability in trajectory height. While many numbers descend quickly, there is no global supremum (*sup*) to limit the maximum value a sequence can attain. This absence of a bound is established by the fact that iterations can be calculated to increment indefinitely based on the structure of the input.

Calculating Infinite Increments of Collatz shortcut for $n = 2^k - 1$

The non-existence of a global supremum is best illustrated by the behavior of integers of the form $n = 2^k - 1$. For these structured cases, we can calculate that the trajectory does not decrease during the first k iterations. The growth can be explicitly mapped as follows:

- $f(n) = \frac{3\cdot(2^k-1)+1}{2} = \frac{3\cdot 2^k-3+1}{2} = \frac{3\cdot 2^k-2}{2} = 3 \cdot 2^{k-1} - 1 > 2 \cdot 2^{k-1} - 1 = 2^k - 1 = n$
- $f^2(n) = 3^2 \cdot 2^{k-2} - 1 > n$
- ...
- $f^k(n) = 3^k - 1 > n$

As $k \to \infty$, these calculated iterations guarantee that the sequence increments to infinity, ensuring the trajectory reaches values significantly higher than its starting point. Because k is an arbitrary parameter that can be increased without limit, the sequence effectively evades any fixed maximum height.

Implications for Trajectory Height

By calculating these iterative steps, it becomes clear that there is no universal constant that can act as a supremum for all Collatz orbits. In these structured cases, the sequence grows according to the power of 3 relative to the power of 2 provided by the input. This ensures that for any proposed upper bound M, there exists a k such that $3^k - 1 > M$, proving that the "up-spikes" in the Collatz sequence are unbounded.

Conclusion

The ability to calculate sustained growth for $n = 2^k - 1$ confirms that there is no universal supremum for the Collatz conjecture. These initial steps guarantee that the trajectory can increment to any arbitrary magnitude as k approaches infinity, making a global upper bound impossible.

References

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